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The Faculty of Power and Aeronautical Engineering  
Division of Theory of Machines and Robots

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Outline

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2. The Faculty of Power and Aeronautical Engineering
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4. Research areas
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Warsaw University of Technology

History:

- First multidisciplinary university of technology in Poland
- **4th January 1826** - opening as Preparatory School for the Institute of Technology
- **1831** - the school was closed after November Insurrection
- **1898** - re-born as the Emperor Nicolas II University with Russian as the language of instruction
- **1905-1909** - the university was closed because of protests in the Kingdom of Poland
- **5th August 1915** - Graman occupants allowed the Warsaw University of Technology to open with Polish as the language of instruction
- **15th August 1915** - opening of the Warsaw University of Technology
- **1967** - The Academic and Research Centre in Plock was created
Warsaw University of Technology

Basic information:

- 32 fields of study
- 19 faculties
- 33,125 students
- 1,037 PhD students
- 1,931 postgraduate students
- 2,486 academic teachers
- 39 buildings for scientific and didactic activities
The Faculty of Power and Aeronautical Engineering

Basic information

- 1960 - year of founding
- Fields of study
  - with Polish as a language of instruction:
    - Automatics and Robotics
    - Power Engineering
    - Aviation and Space Technology
    - Mechanics and Machine Design
  - with English as a language of instruction:
    - Aerospace Engineering
    - Power Engineering
The Faculty of Power and Aeronautical Engineering

Structure

Institute of Aeronautics and Applied Mechanics
- Division of Aerodynamics
- Division of Automation and Aeronautical Systems
- Division of Mechanics
- Division of Fundamentals of Machine Design
- Division of Aeroplanes and Helicopters
- Division of Theory of Machines and Robots
- Division of Strength of Materials and Structures

Institute of Heat Engineering
- Division of Thermodynamics
- Division of Refrigeration and Process Equipment
- Division of Aeroengines
- Division of Power Engineering
- Division of Pumps, Drives and Plants

IAAM

IHE
Basic information:

- **Staff**
  - Head of the Division
  - Secretary
  - 11 researchers
  - 2 technical employees
  - 6 PhD students

- **Research areas**
  - Robotics
  - Biomechanics
  - Modeling and simulation of multibody systems
Robotics

- Design and prototyping of novel technological robots
  - robots for cardio-surgery
  - mobile robots
  - walking machines
  - autonomous boat
Biomechanics

- Biomechanics of impacts
- Hip implants
- Prosthesis
- Rehabilitation devices
- Numerical models of airbags
- Muscle forces
Multibody systems

- Modeling and simulation of rigid multibody dynamics with friction and contact forces in the presence of redundant constraints
- Modeling of flexible multibody systems undergoing small and large deformations
- Co-simulation, coupling techniques for the simulation of multibody dynamics subsystems
- Sequential and parallel algorithms for the simulation of complex multibody systems dynamics
- Research and industrial applications (tracked vehicles, parallel robots, multi-legged machines, railway vehicles, biomechanical systems, molecular systems).
Equipment:

- 2 LWR4+ Robots (7 DOF each) from KUKA
- Agilus Robot (6 DOF) from KUKA
- M1iA Parallel Robot (6 DOF) from FANUC
- M10iA Robot with positioner (6+2 DOF) from FANUC
- belt conveyor
- details sets
- 2D vision system
- Seekur Jr Mobile Robot with 6 DOF manipulator from Adept MobileRobots
- Pioneer P3-DX Mobile Robot with 6 DOF manipulator from Adept MobileRobots
- 3D Scanner
- sensors and measuring device
- safety systems
Student Robotics Association
Comparison of DAE integration methods for multibody dynamics

- Master Thesis under supervision of Prof. Janusz Fraczek, 2012
- Based on: Lorenzo Mariti, Ettore Pennestri, Pier Paolo Valentini, Nicola P. Belfiore; *Review and Comparison of Solution Strategies for Multibody Dynamics Equations*; The 1st Joint International Conference on Multibody System Dynamics; May 25-27; 2010; Lappeenranta; Finland.
- Redundant set of coordinates - the reference point coordinates
Comparison of DAE integration methods for multibody dynamics

The thesis have been made to compare the following methods:

- Orthogonalization methods
  - Zero-eigenvalue method
  - PUTD method (2 types: with Housholder decomposition and Gauss elimination)
  - Schur decomposition
  - SVD decomposition
  - QR decomposition

- Other methods
  - So-called 'classical' method
  - Coordinate partitioning method
  - Least-squares block solution (2 types)
  - Udwadia-Kalaba formulation
  - Udwadia-Phohomsiri formulation
  - Wang-Huston formulation
Comparison of DAE integration methods for multibody dynamics

Algebraic-Differential Equations

\[
M\ddot{q} \pm \Phi_q^T \lambda = Q
\]  \hspace{1cm} (1)

Constraint equations

\[
\Phi(q, t) = 0
\]  \hspace{1cm} (2)

\[
\Phi_q \dot{q} + \Phi_t = 0
\]  \hspace{1cm} (3)

\[
\Phi_q \ddot{q} = -(\Phi_q \dot{q})_q \dot{q} - 2\Phi_q t \dot{q} - \Phi_{tt} = \Gamma
\]  \hspace{1cm} (4)

or:

\[
\Phi_q \ddot{q} = \Gamma - 2\alpha(\Phi_q \dot{q} + \Phi_t) - \beta^2 \Phi = \bar{\Gamma}
\]  \hspace{1cm} (5)

DAE index 1

\[
\begin{bmatrix}
M & \pm \Phi_q^T \\
\Phi_q & 0
\end{bmatrix}
\begin{bmatrix}
\ddot{q} \\
\lambda
\end{bmatrix} =
\begin{bmatrix}
Q \\
\Gamma
\end{bmatrix}
\]  \hspace{1cm} (6)
Comparison of DAE integration methods for multibody dynamics

Orthogonalization methods

And it is assumed that

\[ \dot{v} = B\dot{q} \]  

(7)

\( v \) - independent coordinates vector

We can write it with (3) in the form

\[
\begin{bmatrix}
\Phi_q \\
B
\end{bmatrix}
\dot{q} =
\begin{bmatrix}
-\Phi_t \\
\dot{v}
\end{bmatrix}
\]

(8)

After differentiation

\[
\begin{bmatrix}
\Phi_q \\
B
\end{bmatrix}
\ddot{q} =
\begin{bmatrix}
\Gamma \\
\ddot{v}
\end{bmatrix}
\]

(9)
Comparison of DAE integration methods for multibody dynamics

Orthogonalization methods

From (8)

\[ \dot{q} = \begin{bmatrix} \Phi_q \\ B \end{bmatrix}^{-1} \begin{bmatrix} -\Phi_t \\ \dot{v} \end{bmatrix} = [S \ P] \begin{bmatrix} -\Phi_t \\ \dot{v} \end{bmatrix} = -S\Phi_t + P\dot{v} \]  
(10)

And

\[ \begin{bmatrix} \Phi_q \\ B \end{bmatrix} \begin{bmatrix} \Phi_q \\ B \end{bmatrix}^{-1} = \begin{bmatrix} \Phi_q \\ B \end{bmatrix} \begin{bmatrix} S \ P \end{bmatrix} = \begin{bmatrix} \Phi_qS \\ BS \end{bmatrix} \begin{bmatrix} \Phi_qP \\ BP \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} \]  
(11)

Hence the orthogonalization conditions are

\[ \Phi_qP = 0 \]  
(12)

\[ BP = I \]  
(13)
Comparison of DAE integration methods for multibody dynamics

Orthogonalization methods

From (9)

\[
\ddot{q} = \left[ \begin{array}{c} \Phi_q \\ \mathbf{B} \end{array} \right]^{-1} \left[ \begin{array}{c} \Gamma \\ \ddot{\mathbf{v}} \end{array} \right] = \left[ \begin{array}{cc} \mathbf{S} & \mathbf{P} \end{array} \right] \left[ \begin{array}{c} \Gamma \\ \ddot{\mathbf{v}} \end{array} \right] = \mathbf{S}\Gamma + \mathbf{P}\ddot{\mathbf{v}}
\]

(14)

Where

\[
\mathbf{S}\Gamma = \left[ \begin{array}{c} \Phi_q \\ \mathbf{B} \end{array} \right]^{-1} \left[ \begin{array}{c} \Gamma \\ \mathbf{0} \end{array} \right]
\]

(15)

From (1)

\[
\mathbf{P}^T \mathbf{M}(\mathbf{S}\Gamma + \mathbf{P}\ddot{\mathbf{v}}) \pm \mathbf{P}^T \Phi_q^T \mathbf{\lambda} = \mathbf{P}^T \mathbf{Q}
\]

(16)

Hence

\[
\mathbf{P}^T \mathbf{M}\mathbf{P}\ddot{\mathbf{v}} = \mathbf{P}^T \mathbf{Q} - \mathbf{P}^T \mathbf{M}\mathbf{S}\Gamma
\]

(17)
Comparison of DAE integration methods for multibody dynamics

Other methods

- So-called 'classical' method

\[
\begin{bmatrix}
\ddot{q} \\
\lambda
\end{bmatrix} = \begin{bmatrix}
M & \pm \Phi_q^T \\
\Phi_q & 0
\end{bmatrix}^{-1} \begin{bmatrix}
Q \\
\Gamma
\end{bmatrix}
\]  

(18)

- Coordinate partitioning method (\(u\) - dependent coordinates, \(v\) - independent coordinates)

\[
\ddot{\bar{q}} = \begin{bmatrix}
\ddot{u} \\
\ddot{v}
\end{bmatrix} = \begin{bmatrix}
\Phi_u^{-1}(\Gamma - \Phi_v \ddot{v}) \\
\bar{M}^{-1} \bar{Q}
\end{bmatrix}
\]  

(19)

Where

\[
\bar{M} = \left[ M^{vv} - M^{vu}\Phi_u^{-1}\Phi_v - \Phi_v^T (\Phi_u^{-1})^T (M^{uv} - M^{uu}\Phi_u^{-1}\Phi_v) \right]
\]  

(20)

\[
\bar{Q} = \left[ Q^v - M^{vu}\Phi_u^{-1}\Gamma - \Phi_v^T (\Phi_u^{-1})^T (Q^u - M^{uu}\Phi_u^{-1}\Gamma) \right]
\]  

(21)
Comparison of DAE integration methods for multibody dynamics

Other methods

- Least-squares block solution
  - Multibody systems without singular mass matrix
    \[
    Q_{NK1} = \Phi_q M^{-1} \Phi_q^T
    \] (22)
    \[
    H = M^{-1} \Phi_q^T Q_{NK1}^+
    \] (23)
    \[
    \ddot{q} = (I - H\Phi_q)M^{-1}Q + H\Gamma
    \] (24)

- Any multibody systems
  \[
  E = I - \Phi_q^+ \Phi_q
  \] (25)
  \[
  Q_{NK2} = EME^+
  \] (26)
  \[
  \ddot{q} = Q_{NK2}Q + (\Phi_q^+ - Q_{NK2}M\Phi_q^+)\Gamma
  \] (27)
Comparison of DAE integration methods for multibody dynamics

Other methods

- Udwadia-Kalaba formulation

  \[
  D = \Phi_q M^{-\frac{1}{2}}
  \]

  \[
  \ddot{q}_f = M^{-1} Q
  \]

  \[
  \ddot{q} = \ddot{q}_f + M^{-\frac{1}{2}} D^+ (\Gamma - \Phi_q \ddot{q}_f)
  \]

- Udwadia-Phohomsiri formulation (\(C\) - known vector, \(\eta\) - arbitrary vector)

  \[
  \tilde{M} = \begin{bmatrix}
  (I - \Phi_q \Phi_q^+) M \\
  \Phi_q
  \end{bmatrix}
  \]

  \[
  \ddot{q} = \tilde{M}^+ \begin{bmatrix}
  Q + C \\
  \Gamma
  \end{bmatrix} + (I - \tilde{M}^+ \tilde{M}) \eta
  \]
Comparison of DAE integration methods for multibody dynamics

Other methods

- Wang-Huston formulation

\[ \ddot{q} = M^{-1} \Phi_q T (\Phi_q M^{-1} \Phi_q T)^{-1} \Gamma - (M^{-1} \Phi_q T (\Phi_q M^{-1} \Phi_q T)^{-1}) \Phi_q - I)M^{-1}Q \]  \hspace{1cm} (33)

In the place of

\[ (\Phi_q M^{-1} \Phi_q T)^{-1} \]  \hspace{1cm} (34)

One can use

\[ (\Phi_q M^{-1} \Phi_q T)^+ \]  \hspace{1cm} (35)
Comparison of DAE integration methods for multibody dynamics

Examples:

4 examples have been solved

Planar open-chain mechanism
Planar four-bar mechanism modeled as 2 dimensional structure
Planar four-bar mechanism modeled as spatial structure
Spatial slider-crank mechanism
Comparison of DAE integration methods for multibody dynamics

Example results - computational time for the 2nd example:

<table>
<thead>
<tr>
<th>Method</th>
<th>time [sec]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zero-eigenvalue method</td>
<td>0.624</td>
</tr>
<tr>
<td>PUTD method - H</td>
<td>2.059</td>
</tr>
<tr>
<td>PUTD method - G</td>
<td>1.669</td>
</tr>
<tr>
<td>Schur decomposition</td>
<td>2.231</td>
</tr>
<tr>
<td>SVD decomposition</td>
<td>0.5</td>
</tr>
<tr>
<td>QR decomposition</td>
<td>0.468</td>
</tr>
<tr>
<td>So-called 'classical' method</td>
<td>0.468</td>
</tr>
<tr>
<td>Coordinate partitioning method</td>
<td>1.576</td>
</tr>
<tr>
<td>Least-squares block solution - 1</td>
<td>0.499</td>
</tr>
<tr>
<td>Least-squares block solution - 2</td>
<td>0.515</td>
</tr>
<tr>
<td>Udwadia-Kalaba formulation</td>
<td>0.421</td>
</tr>
<tr>
<td>Udwadia-Phohomsiri formulation</td>
<td>0.686</td>
</tr>
<tr>
<td>Wang-Huston formulation</td>
<td>0.358</td>
</tr>
</tbody>
</table>
Comparison of DAE integration methods for multibody dynamics

Conclusions:

- The methods have different efficiency
- Implementation is an important factor for gaining computational efficiency
- Some of the aforementioned methods do not work with singular mass matrices.
- Wang-Huston formulation is the fastest method for all the examples
PhD studies

- A recursive formulation for constrained multibody system
- Divide and conquer algorithm
- Work in the Robotics Laboratory
This work has been supported by the European Union in the framework of European Social Fund through the Didactic Development Program of the Faculty of Power and Aeronautical Engineering of the Warsaw University of Technology.
Sources:

For more info see:

- http://www.pw.edu.pl
- http://www.meil.pw.edu.pl
- http://tmr.meil.pw.edu.pl
- https://www.facebook.com/mechaniczny.energetyki.i.lotnictwa
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